Period 1, Feb 19, 2025

1. $\int \frac{\sec x \tan x}{\sqrt[4]{\sec x}}\,dx$

$U = \sec x$

$\frac{dU}{\sec x \tan x} = \frac{\cancel{\sec x \tan x}\, dx}{\cancel{\sec x \tan x}}$

$$\frac{dU}{\sec x \tan x} = dx$$

$$\int \frac{\cancel{\sec x \tan x}}{\sqrt[4]{u}} \cdot \frac{dU}{\cancel{\sec x \tan x}} = \int \frac{1}{\sqrt[4]{u}}\,dU = \int U^{-\frac{1}{4}}\,dU = \frac{4}{3} \cdot U^{-\frac{1}{4}+1=\frac{3}{4}} + C$$

$$\frac{4}{3}(\sec x)^{3/4} + C = \frac{4\sec^{\frac{3}{4}} x}{3} + C \neq \frac{4\sec x^{\frac{3}{4}}}{3} + C$$

$(\sec x)^{3/4} = \sec^{\frac{3}{4}} x$

$\cancel{\sec x^{\frac{3}{4}} = \sec(x^{\frac{3}{4}})}$

$\int \frac{\csc x \cot x}{\sqrt[4]{\csc x}}\,dx$

$U = \csc x$

$dU = -\csc x \cot x\, dx$

$$\frac{dU}{-\csc x \cot x} = dx$$

$$\int \frac{\cancel{\csc x \cot x}}{\sqrt[4]{U}} \cdot \frac{dU}{-\cancel{\csc x \cot x}} = \int \frac{-dU}{\sqrt[4]{U}} = -\int U^{-\frac{1}{4}}\,dU = -\cdot\frac{4}{3}\cdot U^{-\frac{1}{4}+1=\frac{3}{4}}$$

$$\frac{-4\csc^{\frac{3}{4}} x}{3} + C$$

$U = 1 + x^4$

$dU = 4x^3\,dx$

$\frac{dU}{4x^3} = dx$

$\int \frac{2x}{U} \cdot \frac{dU}{4x^{\cancel{3}\,2}}$

② $\int \frac{2x}{1+x^4}dx$

$U = x^2$ $\qquad U^2 = x^4$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}+c$

$\int \frac{\cancel{2x}}{1+u^2}\cdot\frac{du}{\cancel{2x}} = \int \frac{du}{1^2+u^2} = \frac{1}{1}\arctan\frac{u}{1}+C = 1\arctan x^2 + C$

$a=1$

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②

$\int \frac{e^x}{4-3e^x}dx$

$U = 4-3e^x$

$du = 0 - 3e^x dx$

$\frac{du}{-3e^x} = dx$

$\int \frac{\cancel{e^x}}{u}\cdot\frac{du}{-3\cancel{e^x}} = -\frac{1}{3}\int\frac{du}{u} = \frac{-1}{3}\ln|u|+C = -\frac{1}{3}\ln|4-3e^x|+C$

$\int u^{-1}du = \ln|u| \qquad = \ln\left|\frac{1}{\sqrt[3]{4-3e^x}}\right|+C$

$\int \frac{-2x}{1+x^4}dx =$

$U = x^2$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$\int \frac{-\cancel{2x}}{1^2+u^2}\cdot\frac{du}{\cancel{2x}}$

$\int \frac{-du}{1^2+u^2} = -\frac{1}{1}\arctan\frac{u}{1}+C \Rightarrow -\arctan\frac{x^2}{1}+C$

$$\int 5x^3(4-7x^4)^{12}\,dx$$

$u = 4 - 7x^4$

$du = 0 - 7 \cdot 4 \cdot x^{4-1}\,dx \Rightarrow du = -28x^3\,dx$

$\frac{du}{-28x^3} = dx$

$$\int 5\cancel{x^3} \cdot u^{12} \cdot \frac{du}{-28\cancel{x^3}} = \int \frac{5u^{12}\,du}{-28} = -\frac{5}{28}\int u^{12}\,du = \frac{-5}{28} \cdot \frac{1}{13} \cdot u^{12+1=13} + C$$

$$\frac{-5}{28 \cdot 13}(4-7x^4)^{13} + C$$

$$\frac{-5(4-7x^4)^{13}}{364} + C$$

$$\int 7x^3(9-4x^4)^{11}\,dx$$

$u = 9 - 4x^4$

$du = 0 - 4 \cdot 4x^{4-1}\,dx$

$du = -16x^3\,dx$

$\frac{du}{-16x^3} = dx$

$$\int 7\cancel{x^3} \cdot u^{11} \cdot \frac{du}{-16\cancel{x^3}}$$

$$\int -\frac{7u^{11}}{16}\,du = -\frac{7}{16}\int u^{11}\,du = -\frac{7}{16} \cdot \frac{1}{12} \cdot u^{11+1=12} + C$$

$$\frac{-7}{16 \cdot 12}(9-4x^4)^{12} + C$$

$$\frac{-7(9-4x^4)^{12}}{192} + C$$

$F(1)=8$

$F'(x) \le 4$

$\underline{(1, 8)}$

Slope is less Then 4

Find Line where Slope $= 4$

Slope less Than 4

$y=4x+b$

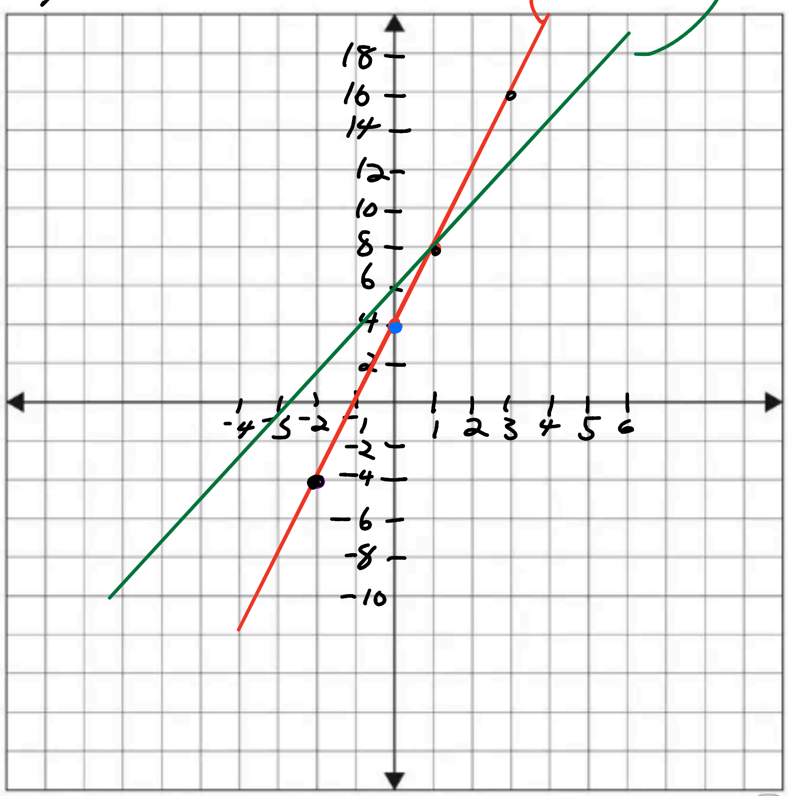
$y=4x+4$

Slope $=4$

$8=4(1)+b$

$8=4+b$

$4=b$



$F(x)=4x+4$

$F(3)=4(3)+4=12+4$

$=16$

$F(3)\le 16$

$F(-2)=4(-2)+4$ Line

$-8+4=-4$

$F(-2)\ge -4$

$F(0)=4$ Line

$F(0)\ge 4$ Line Slope less Than 4

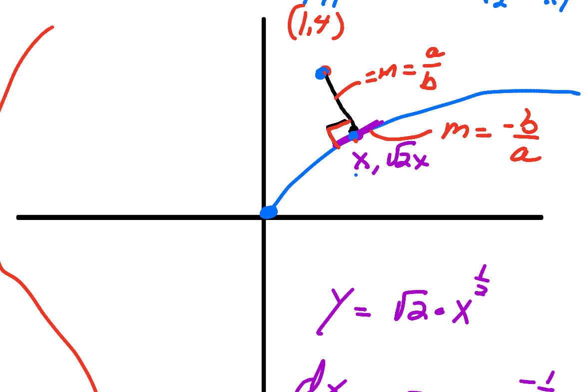
18. $y=\sqrt{2x}$ nearest to $(1,4)$

$m=\frac{y_2-y_1}{x_2-x_1}$

$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$d^2=\left(\sqrt{(\sqrt{2x}-4)^2+(x-1)^2}\right)^2$

To find max/min $\frac{dd}{dx}=0$



$y=\sqrt{2}\cdot x^{\frac{1}{2}}$

$\frac{dy}{dx}=\sqrt{2}\cdot\frac{1}{2}x^{-\frac{1}{2}}$

$\frac{dy}{dx}=\frac{\sqrt{2}}{2\sqrt{x}}$ = slope

$m=\frac{\sqrt{2x}-4}{x-1}$

– Flip

$\frac{-2\sqrt{x}}{\sqrt{2}}=\frac{\sqrt{2x}-4}{x-1}$

$d^2=(\sqrt{2x}-4)^2+(x-1)^2$

$d^2=\cancel{2x}-8\sqrt{2x}+16+x^2\cancel{-2x}+1$

$d^2=-8\sqrt{2x}+17+x^2=(-8\sqrt{2})x^{\frac{1}{2}}+17+x^2$

$2d\frac{dd}{dx}=-8\sqrt{2}\cdot\frac{1}{2}x^{\frac{1}{2}-1=-\frac{1}{2}}+0+2x$

$2\cdot d\cdot 0=\frac{-4\sqrt{2}}{\sqrt{x}}+2x \Rightarrow 0=\frac{-4\sqrt{2}}{\sqrt{x}}+2x \Rightarrow \sqrt{x}\cdot\frac{4\sqrt{2}}{2\cdot\cancel{\sqrt{x}}}=\frac{\cancel{2}x\cdot\sqrt{x}}{\cancel{2}}$

$2\sqrt{2}=x^{3/2}$

$\left(2^{\frac{3}{2}}\right)^{\frac{2}{3}}=\left(x^{3/2}\right)^{\frac{2}{3}}$

$2=x$

$\frac{-2\sqrt{x}}{\sqrt{2}}=\frac{\sqrt{2x}-4}{x-1}$

$(\sqrt{2x}-4)\sqrt{2}=-2\sqrt{x}(x-1)$

$\sqrt{2}\cdot\sqrt{2x}-4\sqrt{2}=-2x\sqrt{x}+2\sqrt{x}$

$2\sqrt{x}-4\sqrt{2}=-2x\sqrt{x}+2\sqrt{x}$

$$\frac{-4\sqrt{2}}{-2} = \frac{-2x\sqrt{x}}{-2} + \cancel{2\sqrt{x}} - \cancel{2\sqrt{x}}$$

$$2\sqrt{2} = x\sqrt{x}$$

$$2^{3/2} = x^{3/2} \Rightarrow x = 2$$

$x^2 + 6^2 = 144$

$x^2 + 36 = 144$

$x^2 = 108$

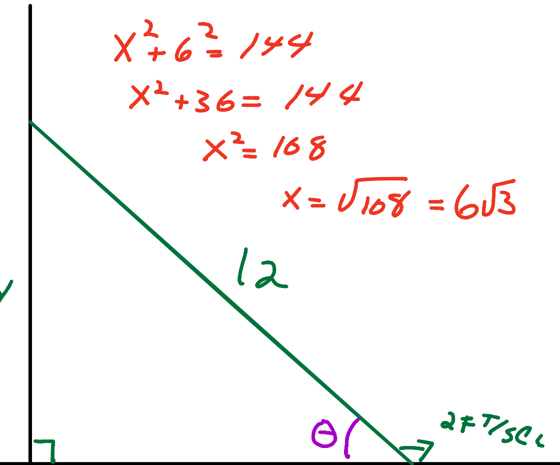
$x = \sqrt{108} = 6\sqrt{3}$

21 (a) $\frac{dy}{dT} = ?$

$y = 6$

$x = 6\sqrt{3}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{12}{x}$



$\cos\theta = \frac{x}{12} = \frac{1}{12}\cdot x$

$-\sin\theta \frac{d\theta}{dT} = \frac{1}{12}\frac{dx}{dT}$

$2\ \text{FT/sec} = \frac{dx}{dT}$

$-\frac{y/12}{12} \frac{1}{12}\frac{dx}{dT} \Rightarrow \frac{-6}{12}\frac{d\theta}{dT} = \frac{1}{12}\cdot 2 \Rightarrow -\frac{1}{2}\frac{d\theta}{dT} = \frac{1}{6} \Rightarrow \frac{d\theta}{dT} = -2\cdot\frac{1}{6} = -\frac{1}{3}$

$x^2 + y^2 = 12^2$

$x^2 + y^2 = 144$

$2x\frac{dx}{dT} + 2y\frac{dy}{dT} = 0$

$2\cdot 6\sqrt{3}\cdot 2 + 2\cdot 6\cdot\frac{dy}{dT} = 0$

$\frac{\cancel{12}\frac{dy}{dT}}{\cancel{12}} = \frac{-24\sqrt{3}}{12} \Rightarrow \frac{dy}{dT} = -2\sqrt{3}\ \text{FT/sec}$

$\tan\theta = \frac{y}{x}$

$\sec^2\theta\,\frac{d\theta}{dT} = \frac{\frac{dy}{dT}\cdot x - y\cdot\frac{dx}{dT}}{x^2}$

$\left(\frac{12}{x}\right)^2\frac{d\theta}{dT} = \frac{-2\sqrt{3}\cdot x - y\cdot 2}{x^2}$

$\left(\frac{12}{\sqrt{108}}\right)^2\frac{d\theta}{dT} = \frac{-2\sqrt{3}\cdot 6\sqrt{3} - 6\cdot 2}{(\sqrt{108})^2}$

$\frac{144}{108}\frac{d\theta}{dT} = \frac{-36-12}{108} = \frac{-48}{108}$

$\frac{\cancel{108}}{144}\cdot\frac{144}{108}\frac{d\theta}{dT} = \frac{-48}{\cancel{108}}\cdot\frac{\cancel{108}}{144}$

$\frac{d\theta}{dT} = \frac{-48}{144} = \frac{-\cancel{12}\cdot 4}{\cancel{12}\cdot 12} = -\frac{4}{12} = \frac{-1}{3}$

Profit $= R(x) - C(x)$

$$P(x) = 90x - \left[0.0002x^3 - 0.1x^2 + 20x + 6000\right]$$

$$P(x) = 90x - 0.0002x^3 + 0.1x^2 - 20x - 6000$$

$$P'(x) = 90 - 0.0002 \cdot 3x^{3-1} + 0.1 \cdot 2x^{2-1} - 20 - 0$$

$$P'(x) = 70 - 0.0006x^2 + 0.2x \Rightarrow 0 = 70 - 0.0006x^2 + 0.2x$$

$$P''(x) = 0 - 0.0012x + 0.2$$

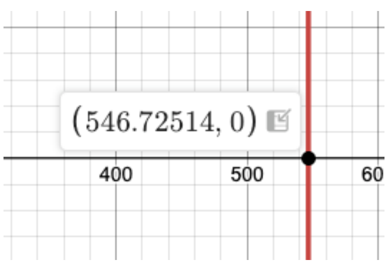
$$0 = -0.0006x^2 + 0.2x + 70$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = -0.0006$

$b = 0.2$

$c = 70$



$X = \underline{546.7}$

$$P''(546.7) = -0.0012(546.7) + 0.2$$

$$= -0.65604 + 0.2 = -0.456$$

negative

concave down

Max

$\# \ \cancel{3d\text{-}f}, \cancel{10}, \cancel{9}, \cancel{4}, \cancel{14}, \cancel{8}, \cancel{2}$

6 $\quad g(x) = \dfrac{(x+1)^{2}(x-3)}{\cancel{(x+1)}(2x+1)^{2}}$ Hole at $x=-1$

a $\quad 2x+1=0$ V.A. $\quad x = -\frac{1}{2}$

b. $\lim\limits_{x\to\infty} \dfrac{(x+1)(x-3)}{(2x+1)^2} = \dfrac{\cancel{x^2}}{4\cancel{x^2}} = \dfrac{1}{4}$ bill gates Rule

$\lim\limits_{x\to-\infty} \dfrac{x^2}{4x^2} = \dfrac{1}{4}$

c $\quad x \neq -1 \quad x \neq -\frac{1}{2}$

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3. $g(x) = \frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{3}{2}x^2 + 1$

$g'(x) = x^3 - 2x^2 - 3x$

$g''(x) = 3x^2 - 4x - 3$

$52 \to 4 \cdot 13$

$\sqrt{52} = \sqrt{4 \cdot 13} = 2\sqrt{13}$

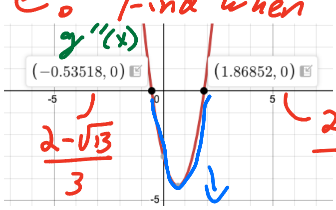
d. Find possible points of inflection $g''(x) = 0$ or ϕ

$0 = 3x^2 - 4x - 3 \qquad x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4\cdot3\cdot(-3)}}{2\cdot3} = \dfrac{4 \pm \sqrt{52}}{6}$

$x = \dfrac{4 \pm 2\sqrt{13}}{6} \Rightarrow x = \dfrac{\cancel{2}(2 \pm \sqrt{13})}{3\cdot\cancel{2}}$

$x = \dfrac{2+\sqrt{13}}{3}$ or $\dfrac{2-\sqrt{13}}{3}$

e. Find when $g(x)$ is concave upward. $g''(x) = +$



$\dfrac{2-\sqrt{13}}{3} \qquad \dfrac{2+\sqrt{13}}{3}$

$\left(-\infty, \dfrac{2-\sqrt{13}}{3}\right) \cup \left(\dfrac{2+\sqrt{13}}{3}, \infty\right)$

F. Find when $g(x)$ is concave downward. $g''(x)=-$

$\left(\frac{2-\sqrt{13}}{3}, \frac{2+\sqrt{13}}{3}\right)$

2. $g(x)=\frac{1}{4}x^4+\frac{2}{3}x^3-\frac{3}{2}x^2+4$

$g'(x)=x^3+2x^2-3x$

$g''(x)=3x^2+4x-3$

ⓐ critical #'s $g'(x)=0$ or $\varnothing \Rightarrow 0=x^3+2x^2-3x=x(x^2+2x-3)$

$g'(x)=x(x+3)(x-1)$

critical #'s

$x=0,-3,1$

ⓑ $g(0)=\frac{1}{4}(0)^4+\frac{2}{3}(0)^3-\frac{3}{2}(0)^2+4=4 \Rightarrow (0,4)$

$g(-3)=\frac{1}{4}(-3)^4+\frac{2}{3}(-3)^3-\frac{3}{2}(-3)^2+4$

$\frac{81}{4}-\frac{54}{3}-\frac{27}{2}+4=-7\frac{1}{4}=\frac{-29}{4} \Rightarrow \left(-3,\frac{-29}{4}\right)$

$g(1)=\frac{1}{4}(1)^4+\frac{2}{3}(1)^3-\frac{3}{2}(1)^2+4$

$\frac{1}{4}+\frac{2}{3}-\frac{3}{2}+\frac{4}{1}=\frac{3}{12}+\frac{8}{12}-\frac{18}{12}+\frac{48}{12}=\frac{41}{12} \Rightarrow \left(1,\frac{41}{12}\right)$ or $\left(1,3\frac{5}{12}\right)$

ⓒ ⓓ Absolute Max/min $[-1,2]$

x	$g(x)$
0	4
1	$3\frac{5}{12}$
-1	$\frac{25}{12}=2\frac{1}{12}$ Min
2	$7\frac{1}{3}$ Max

$g(-1)=\frac{1}{4}-\frac{2}{3}-\frac{3}{2}+4=\frac{3}{12}-\frac{8}{12}-\frac{18}{12}+\frac{48}{12}=\frac{25}{12}$

$g(2)=\frac{1}{4}(2)^4+\frac{2}{3}(2)^3-\frac{3}{2}(2)^2+4$

$4+\frac{16}{3}-6+4=2+\frac{16}{3}=2+5\frac{1}{3}=7\frac{1}{3}$

(e) (f) Absolute Max/min $[-4,1]$

x	$g(x)$
1	$3\frac{5}{12}$
-3	$-3\frac{3}{4}$ Min
0	4 Max
-4	$-1\frac{1}{3}$

$$g(-4) = \frac{1}{4}(-4)^4 + \frac{2}{3}(-4)^3 - \frac{3}{2}(-4)^2 + 4$$
$$= 64 - 42\frac{2}{3} - 24 + 4 = -1\frac{1}{3}$$

15. approx $\sqrt{99.9}$ $\quad \sqrt{100} = 10$

$y = \sqrt{x}$ $\quad$ $\overset{x}{(100}, \overset{y}{10)}$

$y = x^{\frac{1}{2}}$

$dy = \frac{1}{2}x^{\frac{1}{2}-1}\,dx$ $\quad$ $dx = -0.1$, $x = 100$

$dy = \frac{1}{2\sqrt{x}} \cdot (-0.1)$

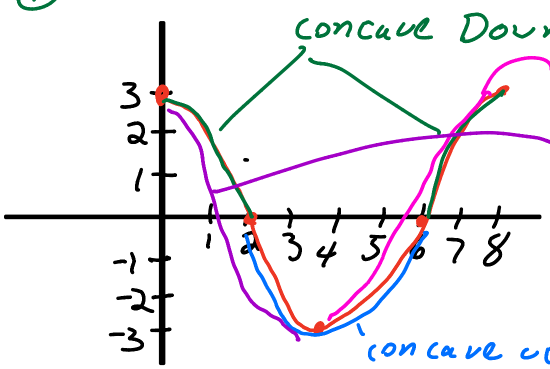
$dy = \frac{1}{2\sqrt{100}}(-0.1)$

Change in y: $dy = \frac{-0.1}{2 \cdot 10} = \frac{-0.1}{20} = -0.005$

Approx $10 - 0.005$

9.995

(4)



concave Down so $F''(x) = -$ so Slope of $F'(x)$ is negative

$F'(x)$ is decreasing

$F'(x) = +$ because $F(x)$ is increasing

$F'(x) = -$ because $F(x)$ is decreasing

a. $(4,6)$ concave up, $F'(x) = +$

b. $(0,2)$ concave DN, $F'(x) = -$

c. $(6,8)$ concave DN, $F'(x) = +$

d. $(2,4)$ concave up, $F'(x) = -$

concave up $F''(x) = +$ so Slope of $F'(x)$ is positive

$F'(x)$ is increasing

8. $F'(x) = (x^2-4)\sin(2.5x)$ $(-3,3)$

$2.5x = \pi$ $\quad$ $2.5x = 2\pi$

$x = \frac{\pi}{2.5}$ $\quad$ $x = \frac{2\pi}{2.5}$

$F'(x) = (x+2)(x-2)\sin 2.5x$

$\sin 2.5x = 0$ when $x = 0, \frac{\pi}{2.5}, x = \frac{2\pi}{2.5}$

$x = 0, 1.257, 2.513,$
$-1.257, -2.513$

$F'(2)=0$

$F'(-2)=0$

$F'(0)=0$

$F'(1.257)=0$

$F'(2.513)=0$

$F'(-1.257)=0$

$F'(-2.513)=0$

a $F(x)$ is increasing

$(-2.513,-2) \cup (-1.257,0) \cup (1.257,2) \cup (2.513,3)$

$F(x)$ is decreasing

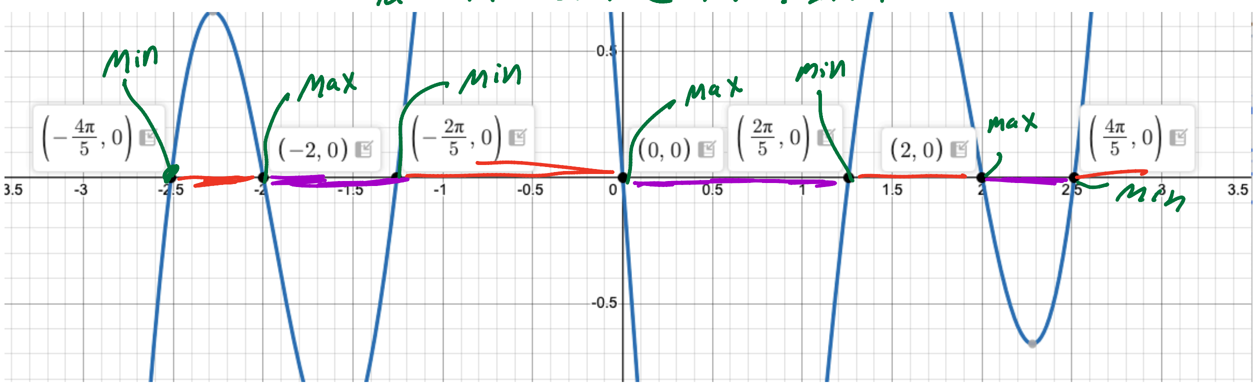
$(-2,-1.257) \cup (0,1.257) \cup (2,2.513)$

Max $F'(x)$ + To −

Min $F'(x)$ − To +

e 7 Extrema

c,d Relative max/min



9 (a) $\underbrace{F'(x)>0 \text{ For all } x}_{\text{F(x) is getting bigger}}$ and $\underbrace{F''(x)>0 \text{ For all } x}_{\text{F(x) is getting Bigger by bigger Margins}}$

x	F(x)
-1	2
1	6
3	13

+4, +7

(b) $\underbrace{F'(x)>0}_{\text{F(x) is getting bigger}}$ and $F''(x)<0$

F(x) is getting bigger by smaller Margins

x	F(x)
-1	2
1	6
3	8

+4, +2

(c) $\underbrace{F'(x)>0}_{\text{F(x) is getting bigger}}$ and $\underbrace{F''(x)=0}_{\text{F(x) is getting bigger by same amount}}$

x	F(x)
-1	2
1	6
3	10

+4, +4

(d) $\underbrace{F'(x)<0}_{\text{getting Smaller}}$ and $\underbrace{F''(x)>0}_{\text{Margins getting bigger}}$

$\underset{\text{Small}}{-7} < \overset{\text{big}}{-4}$

x	F(x)
-1	13
1	6
3	2

-7, -4

(e) $\underbrace{F'(x)<0}_{\text{getting smaller}}$ and $\underbrace{F''(x)<0}_{\text{Margins Smaller}}$

x	F(x)
-1	8
1	6
3	2

-2 Bigger

-4 smaller

$-2 > -4$

(f) $F'(x) < 0$ $\quad$ $F''(x) = 0$

getting smaller $\quad$ Same margins

x	$F(x)$
-1	10
1	6
3	2

$10 \to 6$: -4, $6 \to 2$: -4

(g) $F'(x) = 0$ and $F''(x) = 0$

stays the same

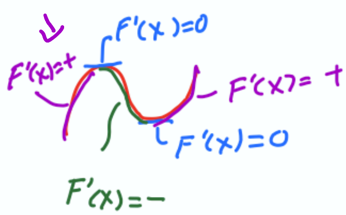
x	$F(x)$
-1	5
1	5
3	5

$5 \to 5$: $+0$, $5 \to 5$: $+0$

#10

$F(x)$ with $F'(x)$ $\qquad$ $F'(x)$

Sample



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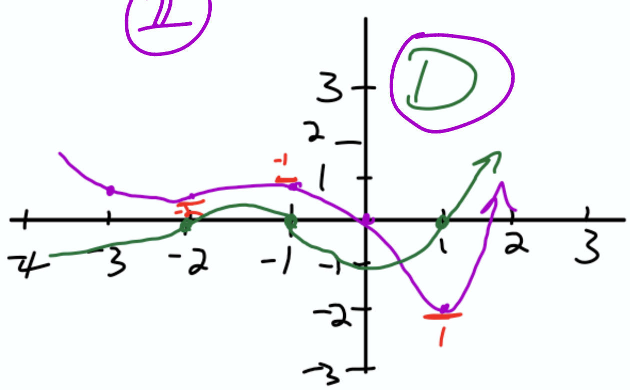

14.

ⓐ $F'(x) = 0 \quad x = -3, -1, 1$

$F'(x) = + \quad (-\infty, -3) \cup (-1, 1)$

$F'(x) = - \quad (-3, -1) \cup (1, \infty)$

ⓑ $F''(x) = 0 \Rightarrow F'(x)$ Slope $= 0 \quad x = -2, 0$

$F''(x) = - \Rightarrow F'(x)$ Slope $= - \quad (-\infty, -2) \cup (0, \infty)$

$F''(x) = + \Rightarrow F'(x)$ Slope $= + \quad (-2, 0)$

ⓒ F is increasing $F'(x) = + \quad (-\infty, -3) \cup (-1, 1)$

ⓓ F is decreasing $F'(x) = - \quad (-3, -1) \cup (1, \infty)$

ⓕ. F has Relative min $F'(x)$ goes From $-$ To $+$

$x = -1$

ⓔ F has Relative Max $F'(x)$ goes From $+$ To $-$

$x = -3, x = 1$

ⓖ F is concave UP, $F''(x) = +$, Slope of $F'(x) = +$

$(-2, 0)$

ⓗ. F is concave Down, $F''(x) = -$, Slope of $F'(x) = -$

$(-\infty, -2) \cup (0, \infty)$